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Optimal Low-Thrust Intercept/Rendezvous Trajectories to Earth-Crossing Objects

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Optimal trajectories are presented that intercept and rendezvous with Earth-crossing objects by using an advanced magnetoplasma spacecraft with variable low-thrust capability. A detailed optimization method is formulated to provide optimal trajectories for any threat scenarios caused by Earth-crossing asteroids/comets. The characteristics of the trajectories are analyzed to find insights into appropriate trajectories for a targeted celestial body. It is primarily illustrated that the optimal trajectories are significantly dependent on the orbital elements of the impacting object, as well as on the distance from the Earth to the target. The trajectories can be used as reference trajectories to formulate an overall concept for solving the deflection problem of the Earth-crossing objects.

Introduction

THE interest in Earth-crossing objects (ECO) has significantly increased since the crash of the comet Shoemaker-Levy 9 into Jupiter in July 1994. It is estimated that there are at least 1000 Earth-crossing asteroids/comets capable of precipitating a global environmental catastrophe on Earth impact.[1] Many previous studies on this subject have primarily concentrated on the detection problem, assessing the magnitude of the threat,[2] analyzing impact effects and hazards to Earth,[3] and conducting a ΔV analysis[4–6] to alter the object's orbit. Despite an increase in interest on the impact mitigation problem, little trajectory analysis has been performed; in particular, the fast intercept/rendezvous trajectory to the dangerous celestial bodies has received scant attention. Here, we further the understanding of one of these problems by formulating and solving the trajectory optimization problem. This paper presents fast intercept/rendezvous spacecraft trajectories to Earth-crossing asteroids/comets. Fast space trips are important to intercept and rendezvous with an impacting asteroid or comet, particularly those not detected many years in advance. Fast trajectories can shorten space flight times and allow orbit modification efforts to begin earlier. The earlier the orbit modification begins, the lesser the change in velocity (ΔV) required to alter the object's trajectory.[4–6] Many studies[7–11] have included intercepts of asteroids/comets as a major part of their mission to study the nature of asteroids. The missions were treated as science missions, and so the flight time was unconstrained and the costs were lower (delta-V budgets), to focus on obtaining the mission trajectory. However, because a deflection mission requires fast travel to a targeted celestial body, these impulsive trajectories in the science missions are not adequate for deflection missions. Intercept flights to the Toutatis asteroid were practically designed by impulsive thrust, for kinetic impact influence on the asteroid orbit.[12] The research demonstrated that small asteroids less than 300 m in radius could be deflected by means of spacecraft kinetic impact. This research was limited to an impulsive trajectory in two-dimensional space and only to a specified asteroid. Many low-thrust trajectories to other planets have been studied.[13,14] Because asteroids and comets usually have high orbital inclination and high eccentricity,[15] the relative velocity of asteroids and comets with respect to the Earth is quite different from those of planets. Thus, low-thrust trajectories to asteroids and comets are somewhat different from the trajectories for planet explorations. Some space missions, such as Deep Space-1 (Refs. 16 and 17) and SMART-1 (Ref. 18), use low-thrust trajectory to asteroids or comets. These trajectories also do not permit the fast travel that is required for asteroid/comet deflection missions. A collocation method coupled with a nonlinear programming technique outlines a low-thrust intercept trajectory[19] to two specified Earth-crossing asteroids by using current electric propulsion technology. This study showed that the use of low-thrust propulsion could reduce the mass of the interceptor vehicle at launch and showed the usefulness of a direct solution method for asteroid interception. However, for mitigation missions, it is still necessary to investigate insights into fast intercept/rendezvous trajectories with Earth-crossing asteroids/comets. The present paper presents a study of the mitigation trajectories in three-dimensional space by using an advanced spacecraft with variable continuous thrust. This study uses an indirect optimization method for the mitigation trajectories. For various types of ECO, the characteristics of the intercept and rendezvous trajectories are analyzed with respect to spacecraft departure time before collisions.

As new propulsion systems of spacecraft have been developed, trajectories have required new optimization formulations because trajectory optimization depends on the characteristics of the spacecraft. Many future propulsion systems have been proposed and analyzed. One potential propulsion approach that has been examined for deflection capability is the variable specific impulse magnetoplasma rocket (VASIMR). A VASIMR is a high-power magnetoplasma rocket that gives continuous and variable thrust at constant power.[20] Thrust is increased proportionally to the power level. The engine can optimize propellant usage and deliver a maximum payload in minimum time by varying thrust and I_{sp} (Ref. 21). Therefore, a VASIMR can yield the fastest possible trip time with a given amount of propellant by using constant power throttling. A 10-kW space demonstrator experiment has been completed,[22] and a VASIMR engine with 200-MW power could be available around the year 2050. A VASIMR propulsion system also needs a new trajectory optimization technique. Hence, this paper was written to attempt to make a contribution to utilizing the VASIMR propulsion system to achieve fast intercept/rendezvous trajectory to Earth-crossing asteroids/comets. The main goal is to find fast intercept/rendezvous spacecraft trajectories with Earth-crossing asteroids/comets and to investigate overall insights into the trajectories. When an advanced VASIMR is used, optimization problems in three-dimensional space are formulated to minimize flight time with moderate propellant mass. The optimal thrust-vector history and propellant mass to be used are found order to transfer a spacecraft

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from the Earth to a targeted celestial object. Using the insights into intercept/rendezvous trajectory obtained in this paper, we can establish the approximate cost and build strategies to prevent possible catastrophe due to Earth-crossing asteroids/comets. In the first section, the trajectory optimization algorithm is refined with control inequality constraints. The second section focuses on the formulation of optimal intercept/rendezvous trajectories to ECO for VASIMR spacecraft. The results and characteristics of the trajectories are discussed in the third section.

Formulation of Intercept/Rendezvous Problems

Many problems in the design of modern guidance and control systems require optimization of the trajectory that minimizes (or maximizes) some performance criterion. When the theory of the calculus of variations is used, the formulation of these problems yields a two-point boundary-value problem. The resulting optimal trajectory also satisfies the physical constraints and the given differential equations. The open-loop optimal trajectory can be calculated as a reference trajectory for intercept/rendezvous with Earth-crossing asteroids/comets. For the trajectory, the control variables are thrust direction angle in plane, thrust direction angle of out of plane, and specific impulse. Because the specific impulse is bounded, the optimization problem has a control inequality constraint [$\boldsymbol{u}_{\min} \leq \boldsymbol{u}(t) \leq \boldsymbol{u}_{\max}$]. A general optimal control problem with control inequality constraints can be obtained.[23–25] The control variable inequality constraint is adjoined to the cost function, and additional necessary conditions are obtained as a result. The optimal trajectory is composed of two types of control: nominal control [$\boldsymbol{u}_{\min} < \boldsymbol{u}(t) < \boldsymbol{u}_{\max}$] and boundary control [$\boldsymbol{u}(t) = \boldsymbol{u}_{\min}$ or $\boldsymbol{u}_{\max}$]. Nominal control satisfies the same necessary conditions as the unconstrained problem. For boundary control, the inequality constraint becomes an equality constraint. Many classical problems in the calculus of variations treat constraints of this form very well. Pontryagin's minimum principle requires that the optimal controls minimize the Hamiltonian function. The conditions to be satisfied to minimize the augmented performance index are found by taking the first variation of the cost function and setting relations equal to zero. From this, the states $\boldsymbol{x}(t)$, costates $\boldsymbol{\lambda}(t)$, and the augmented Hamiltonian function satisfy the optimality and necessary conditions.

Performance Index

Fast space trips are required to intercept and rendezvous with an impacting asteroid/comet. Fast trajectories can shorten space flight times and allow orbit modification efforts to begin earlier. However, shorter trip times require more propellant to provide enough thrust. This additional propellant mass can be a burden to the architecture of spacecraft because the performance of spacecraft may depend on spacecraft's initial wet mass. Thus, it is necessary to achieve an appropriate balance between flight time and propellant mass. The goal is to minimize flight time with moderate propellant mass. The performance index is described by a weighted sum of the flight time and required propellant. The final mass of spacecraft is set as its dry mass because the propellant is exhausted when the spacecraft meets target celestial bodies. Hence, optimal control theory is concerned with finding the control history to optimize a measure of the performance index of the following general form: Minimize

$$J(u) = h_0[\boldsymbol{x}(t_0)] + h_f[\boldsymbol{x}(t_f), t_f] = m_0 + ct_f \tag{1}$$

where m_0 represents the initial mass of the spacecraft, t_f is the flight time or free final time, t_0 is initial time, and c is weight for flight time. As c is increased, the flight time is decreased; consequently, required propellant is increased. If c is greater than 10, required propellant is dramatically increased to reduce the flight time by only several days. Hence, c is experimentally set as 10 in this paper. For the problem at hand, it is required to determine the optimal trajectory that minimizes the initial mass of the spacecraft and the balanced flight time.

Spacecraft and Propulsion

The spacecraft has a VASIMR that is a high-power magnetoplasma rocket. It is expected that the VASIMR engine could be equipped with a 200-MW power support system available around the year 2050. Hydrogen plasma is heated by rf power to increase exhaust velocity up to 300 km/s. The engine gives continuous and variable thrust at constant power. The power output of the engine is kept constant, thus, thrust and specific impulse I_{sp} are inversely related. The engine can optimize propellant usage and deliver a maximum payload in minimum time by varying I_{sp} (3000–30,000 s). A multimegawatt nuclear electric propulsion (NEP) system utilizing a VASIMR engine is currently estimated to have an overall specific mass of less than 1.0 kg/kW (Ref. 26). For a 200-MW system, this would result in a maximum spacecraft dry mass of 200 mt (neglecting payload mass). This is the total spacecraft mass assumed in this paper. An assumed power efficiency of 60% is reasonable. To calculate acceleration a and spacecraft mass flow rate, the following relationships are used. The thrust, $T = |\dot{m}|v_e$ and exhaust velocity, $v_e = I_{\text{sp}}g_0$ are described by specific impulse I_{sp} and the acceleration due to gravity at the Earth's sea level g_0; $\dot{m}$ is itself a negative value. The power P required to expel mass at the mass flow rate $\dot{m}$ is $\varepsilon P = \frac{1}{2}|\dot{m}|v_e^2$, where ε is the efficiency of the propulsion system (assumed to be 0.6). Thus, trust and power has the following relationship:

$$T = 2\varepsilon P / I_{\text{sp}} g_0 \tag{2}$$

When Eq. (2) is used, acceleration due to VASIMR can be derived

$$a = T/m = (2\varepsilon P / m g_0)(1/I_{\text{sp}}) \tag{3}$$

where m is the spacecraft mass at any time. Finally, the mass flow rate is calculated as

$$\dot{m} = -T/v_e = -\left(2\varepsilon P \big/ g_0^2\right)\left(1 \big/ I_{\text{sp}}^2\right) \tag{4}$$

Equations of Motion

The spacecraft is considered to fly in three-dimensional space in the heliocentric system. The three-degree-of-freedom equations of motion are as follows:

$$\dot{r} = u \tag{5a}$$

$$\dot{u} = \frac{v^2}{r} + \frac{w^2}{r} + a \sin\alpha \cos\beta - \frac{1}{r^2} \tag{5b}$$

$$\dot{v} = -\frac{uv}{r} + \frac{vw \sin\phi}{r \cos\phi} + a \cos\alpha \cos\beta \tag{5c}$$

$$\dot{w} = -\frac{uw}{r} - \frac{v^2 \sin\phi}{r \cos\phi} + a \sin\beta \tag{5d}$$

$$\dot{\theta} = \frac{v}{r \cos\phi} \tag{5e}$$

$$\dot{\phi} = \frac{w}{r} \tag{5f}$$

$$\dot{m} = -\frac{2\varepsilon P}{g^2 I_{\text{sp}}^2} \tag{5g}$$

where r is the radial distance from the sun to the spacecraft, u is the radial velocity, v is the tangential velocity, w is the normal velocity, θ is the angle measured from the x axis (defined as the vernal equinox) in the x–y plane (ecliptic plane), and ϕ is the angle measured from the x–y plane. The control variables are the thrust direction angle in plane α, the thrust direction angle of out of plane β, and the variable specific impulse I_{sp}.

The Hamiltonian function is defined as

$$H = \lambda_r u + \lambda_u \left[\frac{v^2}{r} + \frac{w^2}{r} + a \sin\alpha \cos\beta - \frac{1}{r^2}\right] + \lambda_v \left[-\frac{uv}{r} + \frac{vw \sin\phi}{r \cos\phi} + a \cos\alpha \cos\beta\right] + \lambda_w \left[-\frac{uw}{r} - \frac{v^2 \sin\phi}{r \cos\phi} + a \sin\beta\right] + \lambda_\theta \frac{v}{r \cos\phi} + \lambda_\phi \frac{w}{r} - \lambda_m \frac{2\varepsilon P}{g^2 I_{\text{sp}}^2} \tag{6}$$

When the Hamiltonian function is used, the costate equations are obtained as

$$\dot{\lambda}_r = \lambda_u\left[\frac{v^2+w^2}{r^2} - \frac{2}{r^3}\right] + \lambda_v\left[\frac{vw\sin\phi}{r^2\cos\phi} - \frac{uv}{r^2}\right] + \lambda_w\left[-\frac{v^2\sin\phi}{r^2\cos\phi} - \frac{uw}{r^2}\right] + \lambda_\theta\frac{v}{r^2\cos\phi} + \lambda_\phi\frac{w}{r^2} \tag{7a}$$

$$\dot{\lambda}_u = -\lambda_r + \lambda_v\frac{v}{r} + \lambda_w\frac{w}{r} \tag{7b}$$

$$\dot{\lambda}_v = -\lambda_u\frac{2v}{r} + \lambda_v\left[\frac{u}{r} - \frac{w\sin\phi}{r\cos\phi}\right] + \lambda_w\frac{2v\sin\phi}{r\cos\phi} - \lambda_\theta\frac{1}{r\cos\phi} \tag{7c}$$

$$\dot{\lambda}_w = -\lambda_u\frac{2w}{r} - \lambda_v\frac{v\sin\phi}{r\cos\phi} + \lambda_w\frac{u}{r} - \lambda_\phi\frac{1}{r} \tag{7d}$$

$$\dot{\lambda}_\theta = 0 \tag{7e}$$

$$\dot{\lambda}_\phi = -\lambda_v\frac{vw}{r\cos^2\phi} + \lambda_w\frac{v^2}{r\cos^2\phi} - \lambda_\theta\frac{v\sin\phi}{r\cos^2\phi} \tag{7f}$$

$$\dot{\lambda}_m = \lambda_u\frac{2\varepsilon P}{m^2 g I_{\rm sp}}\sin\alpha\cos\beta + \lambda_v\frac{2\varepsilon P}{m^2 g I_{\rm sp}}\cos\alpha\cos\beta + \lambda_w\frac{2\varepsilon P}{m^2 g I_{\rm sp}}\sin\beta \tag{7g}$$

Initial Conditions and Terminal Conditions

In the future, a space gate could be located at the L_1 Lagrange point of the Earth–moon system.[27] It is reasonable to place the spacecraft in the space gate at the L_1 for asteroid patrol.[28] Thus, the spacecraft is assumed to depart from the L_1 point that is far from the Earth. Consequently, required fuel and flight time to escape the Earth is relatively very small when compared with those for heliocentric trajectory. The intercept/rendezvous trajectories in this study are optimized in only the heliocentric phase because the Earth-escape phase would be negligible. The spacecraft departs with the following initial conditions (at $t = 0$ s): $r(t_0) = 1$, $u(t_0) = 0$, $v(t_0) = 1$, $w(t_0) = 0$, $\theta(t_0) =$ obtained from departure time before collision, $\phi(t_0) = 0$, $m(t_0) =$ free (unknown), and $t_f =$ free (unknown).

The initial mass of the spacecraft is a free parameter to include propellant mass that is needed for fuel of the intercept and rendezvous trajectory. The fundamental theorem of the calculus of variation yields the boundary conditions of the corresponding costates for unknown states at initial time,

$$\lambda(t_0) = -\frac{\partial h_0}{\partial \boldsymbol{x}}\bigg|_{t_0} \tag{8}$$

Consequently, initially the costate of mass is set as $\lambda_m(t_0) = -1$, and the other costate values $[\lambda_r(t_0), \lambda_u(t_0), \lambda_v(t_0), \lambda_w(t_0), \lambda_\theta(t_0), \lambda_\phi(t_0)]$ at t_0 are unknown. The spacecraft must intercept or rendezvous with the targeted asteroid/comet with a specified orbit. To achieve the desired trajectory, final conditions should be satisfied. These are the positions of the spacecraft for the intercept trajectory and the positions and velocities of the spacecraft for the rendezvous trajectory. Hence, for the intercept trajectory, the terminal state conditions are

$$\Phi[x(t_f), t_f] = \begin{Bmatrix} r(t_f) - r_{\rm target}(t_f) \\ \theta(t_f) - \theta_{\rm target}(t_f) \\ \phi(t_f) - \phi_{\rm target}(t_f) \\ m(t_f) - m_{\rm dry} \end{Bmatrix} = 0 \tag{9}$$

where subscript target denotes the state of the targeted celestial object and $m_{\rm dry}$ is the spacecraft dry mass. The rendezvous trajectory has the following terminal state conditions:

$$\Phi[x(t_f), t_f] \equiv \begin{Bmatrix} r(t_f) - r_{\rm target}(t_f) \\ u(t_f) - u_{\rm target}(t_f) \\ v(t_f) - v_{\rm target}(t_f) \\ w(t_f) - w_{\rm target}(t_f) \\ \theta(t_f) - \theta_{\rm target}(t_f) \\ \phi(t_f) - \phi_{\rm target}(t_f) \\ m(t_f) - m_{\rm dry} \end{Bmatrix} = 0 \tag{10}$$

From the transversality conditions, we obtain the following costate terminal conditions for the intercept trajectory:

$$\lambda_r(t_f) = \nu_1, \qquad \lambda_u(t_f) = \lambda_v(t_f) = \lambda_w(t_f) = 0, \qquad \lambda_\theta(t_f) = \nu_2$$
$$\lambda_\phi(t_f) = \nu_3, \qquad \lambda_m(t_f) = \nu_4 \tag{11}$$

where ν_i is the constant Lagrange mutiplier. From transversality conditions, we obtain the following costate terminal conditions for the rendezvous trajectory:

$$\lambda_r(t_f) = \nu_1, \qquad \lambda_u(t_f) = \nu_2, \qquad \lambda_v(t_f) = \nu_3$$
$$\lambda_w(t_f) = \nu_4, \qquad \lambda_\theta(t_f) = \nu_5, \qquad \lambda_\phi(t_f) = \nu_6, \qquad \lambda_m(t_f) = \nu_7 \tag{12}$$

Furthermore, the following condition for the Hamiltonian function is also satisfied at t_f:

$$H(t_f) = -c \tag{13}$$

Equation (13) becomes another boundary equation at the final time.

Controls

The control variables are thrust direction angle in plane α, thrust direction angle of out of plane β, and specific impulse $I_{\rm sp}$. A second-order necessary condition, that is, convexity condition, namely, the Legendre condition, states that the second derivative of the Hamiltonian function with respect to the controls must be greater than or equal to zero for the performance index to be at a minimum. Thus, $\partial^2 H/\partial\alpha^2$ and $\partial^2 H/\partial\beta^2$ must be greater than or equal to zero for the performance index to be at a minimum. The first derivative of H with respect to α and the convexity condition yield the control variable of α, as follows:

$$\sin\alpha = -\lambda_u\Big/\sqrt{\lambda_u^2 + \lambda_v^2} \tag{14a}$$

$$\cos\alpha = -\lambda_v\Big/\sqrt{\lambda_u^2 + \lambda_v^2} \tag{14b}$$

The first derivative of H with respect to β and the convexity condition with Eq. (14) yield the control variable of β, as follows:

$$\sin\beta = \frac{-\lambda_w}{\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{15a}$$

$$\cos\beta = \frac{\sqrt{\lambda_u^2 + \lambda_v^2}}{\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{15b}$$

The first derivative of H with respect to $I_{\rm sp}$ and the convexity condition with Eqs. (14) and (15) yield the control variable of $I_{\rm sp}$, as follows:

$$I_{\rm sp} = \frac{-2m\lambda_m}{g\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{16}$$

$$I_{\rm sp\,min} \le I_{\rm sp} \le I_{\rm sp\,max} \tag{17}$$

If the $I_{\rm sp}$ constraint were not active and $\lambda_m < 0$, the solution would be optimal. If the unconstraint control $I_{\rm sp} < I_{\rm sp\,min}$, then $I_{\rm sp} = I_{\rm sp\,min}$, whereas $I_{\rm sp} = I_{\rm sp\,max}$ if the unconstraint control $I_{\rm sp} > I_{\rm sp\,max}$. For $\lambda_m > 0$, the optimal solution is going to $I_{\rm sp} = I_{\rm sp\,min}$. The optimal trajectories have been computed to maximize the final mass of the vehicle and to minimize the weighted flight time. The optimal problem is a free final time problem to find the three control histories satisfying the state and costate equations and boundary conditions described earlier.

Numerical Implications

Many numerical algorithms to solve optimal control problems have been developed. Indirect methods are theoretically based on Pontryagin's minimum principle, which characterizes the set of optimal states and controls in terms of the solution of a boundary value problem. One of the indirect methods becomes a second-order method and yields solutions of high precision; hence, it is very sensitive to small changes in costate initial conditions. The indirect methods have the associated difficulties caused by instability of the initial value problem for the system of differential equations and by the requirement for good initial guesses for iterative solutions of nonlinear problems. In this paper, a shooting method that is formulated by the indirect method described in the preceding section is used to solve comet/asteroid intercept/rendezvous trajectory problems. For the intercept problem, there are 14 differential equations describing the states [Eq. (5)] and costates [Eq. (7)], and there are 15 unknowns [$m(t_0)$, $\lambda_r(t_0)$, $\lambda_u(t_0)$, $\lambda_v(t_0)$, $\lambda_w(t_0)$, $\lambda_\theta(t_0)$, $\lambda_\phi(t_0)$, t_f, ν_1, ν_2, ν_3, ν_4, $u(t_f)$, $v(t_f)$, and $w(t_f)$] and 15 boundary conditions [$\lambda_m(t_0)$, $r(t_0)$, $u(t_0)$, $v(t_0)$, $w(t_0)$, $\theta(t_0)$, $\phi(t_0)$, Eqs. (9), $\lambda_u(t_f)$, $\lambda_v(t_f)$, $\lambda_w(t_f)$, and Eq. (13)]. For the rendezvous problem, there are 14 differential equations describing the states [Eq. (5)] and costates [Eq. (7)], and there are 15 unknowns [$m(t_0)$, $\lambda_r(t_0)$, $\lambda_u(t_0)$, $\lambda_v(t_0)$, $\lambda_w(t_0)$, $\lambda_\theta(t_0)$, $\lambda_\phi(t_0)$, t_f, ν_1, ν_2, ν_3, ν_4, ν_5, ν_6, and ν_7] and 15 boundary conditions [$\lambda_m(t_0)$, $r(t_0)$, $u(t_0)$, $v(t_0)$, $w(t_0)$, $\theta(t_0)$, $\phi(t_0)$, Eqs. (10), and Eq. (13)]. Thus, the two-point boundary problem can be completely solved with these boundary conditions.

To formulate a shooting method for the problems described in this paper, the numerical techniques and codes in Ref. 23 are adopted and modified. The orbital motions of the Earth and Earth-crossing asteroids/comets are set using the techniques described in Ref. 6. The unknown parameters [ν, t_f, unknown initial costates, and $m(t_0)$] are guessed initially, and the state and costate equations are integrated to t_f by implicating the controls. The differential equations are integrated using a Runge–Kutta–Fehlberg fourth-order scheme with an accuracy of nine digits. The initial and final boundary conditions are treated as nonlinear algebraic functions of the unknowns to be determined. The boundary conditions are evaluated using the state, control, and costate values. An error vector is calculated from the difference between the resultant value of Φ and its nominal value, zero. Corrections to the guessed values and the missing initial condition are taken on the basis of a Newton method such that the error vector is forced to zero. The terminal conditions are assumed to be satisfied if the terminal condition norm is less than 10^{-8}. A trial-and-error strategy is used until good initial costates values are obtained. Once converged results for one case are obtained, the converged values can be used as good initial guesses for slightly different cases. By repeating this homotopy method, we can obtain optimal results of the cases we want to solve. Unless one encounters a local singular event, this homotopy method can guarantee convergence. If one encounters a local singular event, a trial-and-error strategy is again used until good initial costates values are found. The numerical results are confirmed as optimal by verifying that the Hamiltonian is constant with respect to time. For well-documented reasons, heliocentric canonical units were used. In this system, the distance units (DU), time units (TU), and speed units (SU) are 1 DU = 1 astronomical unit (AU) = 1.49596×10^8 km, 1 TU = $1/2\pi$ year = 58.17 days, and 1 SU = 29.80 km/s, respectively.

Results of Intercept/Rendezvous Trajectories

By the use of an advanced VASIMR concept, optimization problems in three dimensions are formulated to minimize flight time with moderate propellant mass, as outlined in the preceding sections. The optimal thrust-vector history and propellant mass required can be calculated to transfer the spacecraft from the Earth to a targeted celestial object. These analyses provide fast intercept/rendezvous spacecraft trajectories with Earth-crossing asteroids and comets. In this paper, we consider some typical Earth-crossing asteroids and comets, although the formulation in this paper is applicable to intercept and rendezvous trajectories for any kind of objects. There are three classes of near earth asteroids: Atens, Apollos, and Amors. Aten-type asteroids have a semimajor axis smaller than 1 AU and an aphelion greater than 0.983 AU, whereas Apollo-type asteroids have a semimajor axis greater than 1 AU and a perihelion smaller than 1.017 AU. Hence, Apollo-type and Aten-type asteroids can have ECO. Amors have orbits that lie completely outside the Earth's orbit (perihelion distance between 1.017 and 1.3 AU), but have the potential to be perturbed into Earth-crossing trajectories. For instance, we consider a fictitious Aten-type asteroid with semimajor axis $a = 0.65$ AU, eccentricity $e = 0.6$, and inclination $i = 20$ deg. The asteroid has an orbital period of about 0.52 years, 0.26-AU perihelion distance, and 1.04-AU aphelion distance. A fictitious Apollo-type asteroid is also considered that has its semimajor axis, eccentricity, and inclination fixed at 1.5 AU (orbital period $\approx$ 1.84 years), 0.5, and 20 deg; respectively. Comets are classified into two types: short-period comets (defined here as having an orbital period $<$200 years) and long-period comets (orbital period $>$200 years). We consider a fictitious short-period comet (SPC) whose orbital elements are given by $a = 4.0$ AU, $e = 0.85$, and $i = 20$ deg. For this example, the comet has orbital period of about 8.0 years, perihelion distance of 0.6 AU, and aphelion distance of 7.4 AU. We also consider a fictitious impacting long-period comet (LPC) whose orbital parameters are given by aphelion distance 0.6 AU, perihelion distance 79.4 AU, and inclination $i = 20$ deg. These orbital parameters yield a semimajor axis $a = 40$ AU, eccentricity $e = 0.985$, and an orbital period of 253 years. The four fictitious celestial objects picked as examples are tuned to have ECO. An advanced spacecraft having a 200-MW VASIMR engine is used to calculate optimal intercept/rendezvous trajectories to the fictitious Earth-crossing asteroids/comets. A multimegawatt NEP system utilizing a VASIMR engine is currently estimated to have a maximum overall specific mass of 1.0 kg/kW (Ref. 26). For a 200-MW system, this would result in a total spacecraft dry mass of 200 t (neglecting payload mass). The power efficiency is assumed to be 60%, which would be optimistic. More capable power generators (gigawatt class) with lower specific masses could provide the power needed to generate more cost-effective trajectories. In the discussions to follow, the intercept/rendezvous trajectories are three-dimensional heliocentric trajectories of the spacecraft from the Earth to the target celestial bodies that have collision orbits with the Earth. The departure time is defined as the time interval between the spacecraft's departure and the celestial object's collision with the Earth. This initialization has the advantage of interpreting departure time as the time interval before impact. The flight time refers to the travel time for the spacecraft from the Earth to the target body, whereas the propellant used refers to the required fuel to make the intercept or rendezvous trajectory possible. There are two categories of impact scenarios: one (impact before perihelion, or preperihelion impact) is that an impact occurs before an asteroid/comet sweeps its perihelion, the other (impact after perihelion, or postperihelion impact) is that an impact occurs after an asteroid/comet passes its perihelion. In this paper, only pre-perihelion collision cases are discussed because both of the cases yield very similar phenomena.

The intercept trajectories are needed for the spacecraft to approach the ECO, to deflect their orbits by using spacecraft kinetic energy, nuclear explosion, etc. The rendezvous trajectories are necessary when the spacecraft is landed on the target celestial body or makes formation flying with it, to operate deflection space missions of the ECO. Figure 1 shows examples of normal intercept and rendezvous trajectories when the spacecraft departs to the SPC at a departure time of 8 months before collision. A rendezvous trajectory is compared with an intercept trajectory for the same departure time and same target celestial body. In Fig. 1, the magnitude of thrust vector shows the relative magnitude and direction of the variable thrust due to the variable specific impulse. Generally, for the intercept trajectory, large thrust is required when the spacecraft starts, and the magnitude is monotonically decreased until interception of the target. The thrust history for a rendezvous trajectory has a

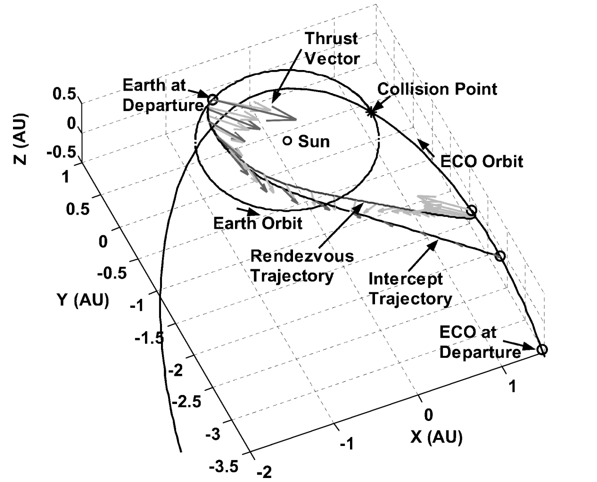

Fig. 1 Intercept and rendezvous trajectories for SPC, magnitude of thrust vector shows relative magnitude and direction of variable thrust.

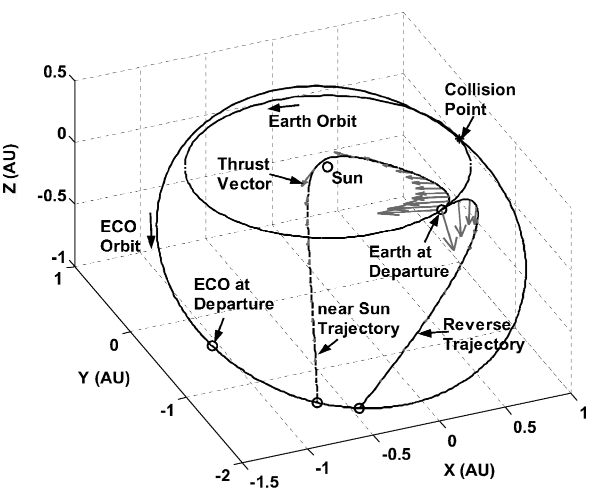

Fig. 2 Near-sun and reverse intercept trajectories for Apollo-type asteroid.

large magnitude during the early part of the trajectory, magnitude decreased in the middle part, and, again, magnitude increased in the late part of the trajectory. The thrust magnitude in the late part is increased because the spacecraft has to match its velocity with the target's orbital velocity. As the departure time from the Earth is varied, optimal trajectory to the given target can sometimes be evolved to abnormal trajectories such as a near-sun trajectory and reverse trajectory. The near-sun trajectory has a path close to the sun, whereas the reverse trajectory has a retrograde path with respect to the Earth's path. Figure 2 shows the case that has both the near-sun trajectory and the reverse trajectory to the Apollo-type asteroid, when the spacecraft departs from the Earth at departure time of 14 months before collision to intercept the target. In Figs. 3 and 4, the propellant required and flight time are shown for intercept trajectories to the Apollo-type asteroid with respect to each departure time. Some departure times yield both the near-sun trajectory and the reverse trajectory. Others yield normal trajectory that is not categorized into the near-sun trajectory or the reverse trajectory. Usually, the reverse trajectories require more fuel expenditure than the near-sun trajectories. When there are two options (near-sun trajectory and the reverse trajectory) for a given departure time, we prefer the trajectory requiring less propellant. The thick solid line in Figs. 3 and 4 shows the propellants required and flight times for the preferred trajectories that require less propellant. For the Aten-type and Apollo-type asteroids and the SPC and LPC described before, Figs. 5 and 6 show collections of the propellants required and of flight times of only preferred intercept trajectories. In figs. 5 and 6, the circle, square, triangle, and diamond symbols mark the propellants required and the flight times for the Aten-type and the

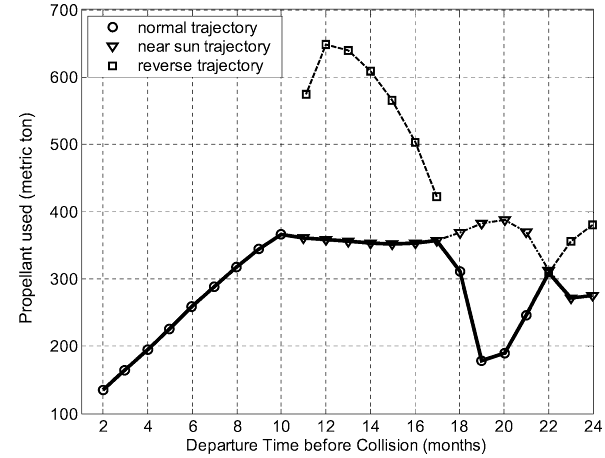

Fig. 3 Required propellant of normal, near-sun, and reverse trajectories for Apollo-type asteroid.

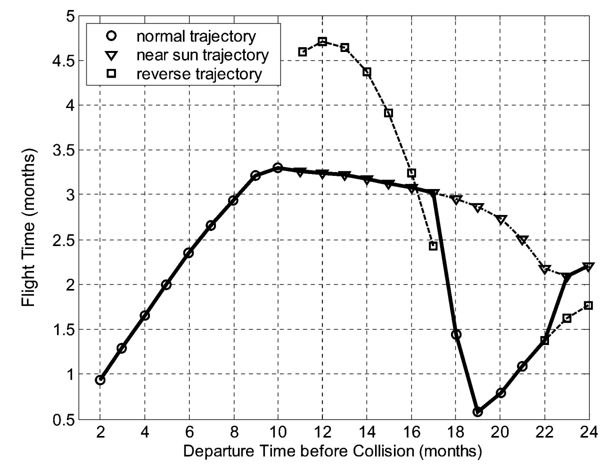

Fig. 4 Flight time of normal, near-sun, and reverse trajectories for Apollo-type asteroid.

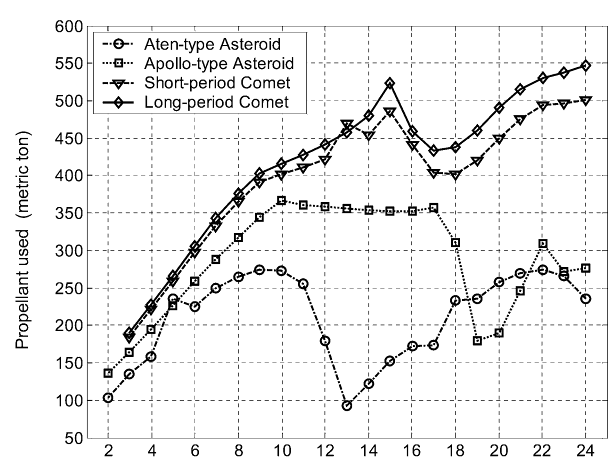

Fig. 5 Required propellant of intercept trajectories for ECO.

Apollo-type asteroids and the SPC and the LPC, respectively. Because there are local minima and maxima in the propellant required and flight time as shown in Figs. 5 and 6, note that the amount of the values are dependent on the orbital geometry relationship, as well as the distance between the Earth and the target. Consequently, sometimes very cost-effective trajectories can be found when the orbital geometry relationship between the Earth and the target is very appropriate for the trajectory. For example, the departure time of 13 months before collision would yield a very cheap intercept trajectory to the Aten-type asteroid, and the departure time of 19

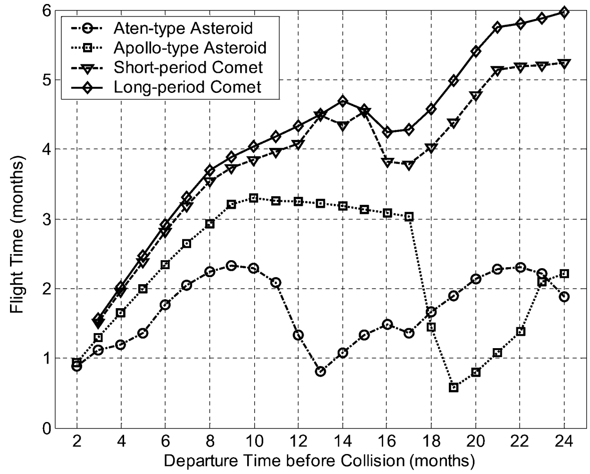

Fig. 6 Flight time of intercept trajectories for ECO.

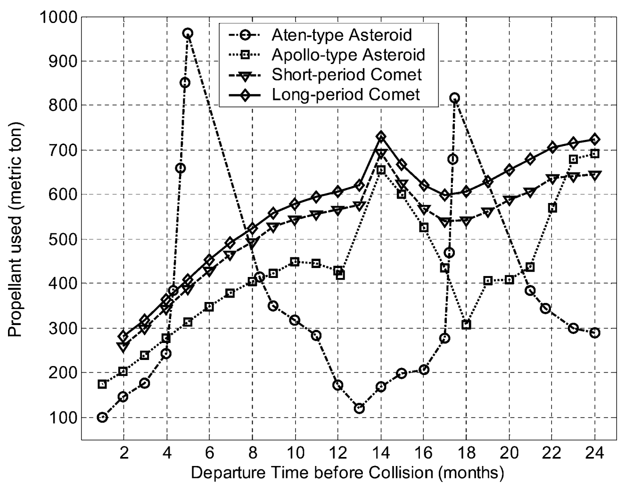

Fig. 7 Required propellant of rendezvous trajectories for ECO.

months before collision would yield a very attractive intercept trajectory to the Apollo-type asteroid. For the comets, there are peaks at 15 months of departure time because the spacecraft must fly in the reverse direction with respect to the Earth's orbital velocity. Generally, for the intercept trajectory, the object with a short orbital period requires relatively less propellant and a shorter flight time. Thus, relatively cheaper (less fuel and shorter flight time) intercept trajectories are acceptable for the celestial targets closer to the Earth. For the ECO described before, Figs. 7 and 8 show the propellants required and flight times of only preferred rendezvous trajectories. In Figs. 7 and 8, the circle, square, triangle, and diamond symbols mark the propellants required and flight times for the Aten-type and the Apollo-type asteroids and the SPC and the LPC, respectively. The rendezvous trajectories also have local minima and maxima in the propellant required and flight time, which prove their dependencies on the orbital geometry relationship and distance between the Earth and the target. For a given departure time, the rendezvous trajectory generally requires more propellant and a longer flight time than the intercept trajectory requires because the velocity of the rendezvous spacecraft must be matched with the target's velocity, which is not required for the intercept trajectory. When the rendezvous spacecraft departs from Earth 12 months before the collision shown in Figs. 7 and 8, the spacecraft can arrive at the Aten-type asteroid (or Apollo-type asteroid, SPC, or LPC) approximately 11 (or 7.6, 6.2, or 5.7) months before impact and requires about 170 (or 420, 570, or 600) mt of propellant. The Aten-type asteroid sometimes needs a lot of propellant (Fig. 7), mainly because its relative geometric position with respect to the Earth position at departure time requires a dramatic reverse trajectory. Except for some special cases, the objects with a longer orbital period usually require larger propellant mass and longer flight time for rendezvous trajectories. When

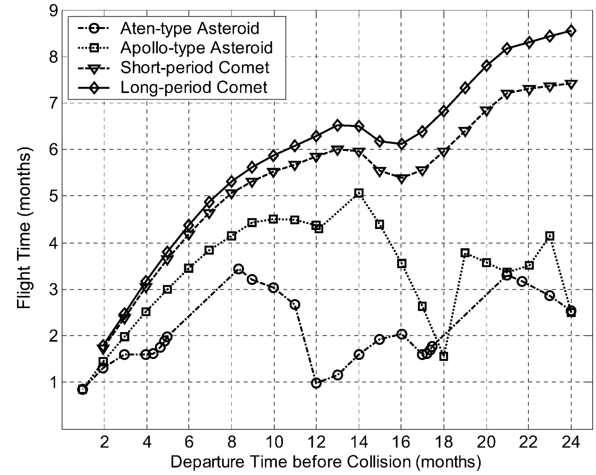

Fig. 8 Flight time of rendezvous trajectories for ECO.

the relative position between the Earth and target is not proper at a given departure time, the rendezvous trajectory forms a reverse, that is, retrograde, path, which requires a large amount of propellant. For comets, the peaks in propellant at a 14-month departure time are good examples for that case. Unlike asteroids, for comets, the distance between the Earth and the target becomes an important parameter contributing to a large amount of propellant and longer flight time.

Earth-crossing asteroids/comets with different orbital elements will also have different flight times and propellant requirements. The differences are caused by the different geometric positions with respect to the target object when the spacecraft departs from the Earth. Even for the same celestial target, a postperihelion impact would have different results from those of a preperihelion impact, mainly because of a different geometric relationship. In case the rendezvous deflection mission was unsuccessful, another spacecraft with a different payload, for example, a nuclear explosive device, could be sent to mitigate the target using an intercept trajectory. Longer warning times would generally reduce the required energy on the orbit modification; hence, early departure would be intuitionally preferred. However, the propellant and flight time for spacecraft trajectory do not linearly depend on the departure time. Consequently, it is challenging to find an appropriate departure time by considering the characteristics of trajectory. For impactors with extremely short warning times, an intercept trajectory may be the only feasible scenario for diverting the object by using a nuclear explosive device. Technological advances that can significantly increase the power of VASIMR spacecraft may permit the intercept/rendezvous trajectory in this paper to become a reality. The VASIMR spacecraft with any other power level would have optimal intercept/rendezvous trajectories with very similar characteristics to those described earlier. A tiered mitigation approach using rapid rendezvous and intercept spacecraft could provide a feasible scenario to protect the Earth from impacting celestial objects.

Conclusions

The optimal intercept and rendezvous trajectories are presented for fictitious target asteroids and comets by using an advanced VASIMR spacecraft concept that is designed to use low thrust. The formulations in this paper can be used to generate intercept/rendezvous trajectories with any celestial objects. Optimal intercept and rendezvous trajectories significantly depend on the geometric positions and distances between the Earth and a targeted ECO for given departure time. Consequently, sometimes very cost-effective trajectories can be found when the orbital geometry relationship between the Earth and the target is very appropriate for the trajectory. The rendezvous trajectory generally requires more propellant and a longer flight time than the intercept trajectory. Generally, the ECO with a short orbital period requires relatively less propellant and a shorter flight time. However, there are no general intercept/rendezvous trajectories to fit all of the ECOs. Thus, it is important to find an appropriate departure time by considering characteristics of the trajectories for each orbit deflection scenario. The required propellant and flight time of the proper trajectories can be used to design an overall mission concept for solving the deflection problem of the Earth-crossing asteroids/comets.

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